ADVANCED ALGEBRA
FINAL EXAM

CHAPTER 5: ALL
CHAPTER 7: LESSONS 7.1-7.5
CHAPTER 8: LESSONS 8.1-8.3, 8.5-8.7
CHAPTER 9: LESSONS 9.1-9.4
CHAPTER 10: LESSONS 10.1-10.4
CHAPTER 11: LESSONS 11.1-11.3
1. The monthly minimum payment \( p \) due on a certain credit card with a fixed rate varies directly as the balance \( b \) and \( p = \$19.80 \) when \( b = \$150.00 \). Find \( p \) when \( b = \$300.00 \) and \( \$24 \).

2. The time \( t \) that it takes Hannah to hike to school varies inversely as her average speed \( s \). If she can hike to school in 25 min when her average speed is 6 mi/h, what would her average speed need to be to get to school in 20 min? 7.5 mi/h.

3. Simplify \( \frac{x^2 - x - 6}{x^2 - 4x + 3} \). Identify any \( x \)-values for which the expression is undefined.

4. Multiply or divide. Assume that all expressions are defined.
   4. \( \frac{x - 5}{2x - 10} \cdot \frac{x - 5}{x^2 - 81} \cdot \frac{1}{2(x + 9)} \)
   5. \( \frac{3x^2 - 9x^2}{x^2 - 16} \div \frac{2x - 6}{x^2 - 8x + 16} \)

   \( \frac{3x^2(x - 4)}{2(x + 4)} \)

5. Add or subtract. Identify any \( x \)-values for which the expression is undefined.

6. \( \frac{5}{x - 3} + \frac{x}{2x - 10} \)

7. \( \frac{5x}{x - 7} - \frac{9x - 8}{x + 3} \)

8. Lorraine averaged 62 words per minute when typing the first 3 pages of a 6-page report. Her average typing speed for the last 3 pages was 45 words per minute. To the nearest word per minute, what was Lorraine’s average typing speed for the entire report? 59 words/min.

9. Identify the zeros and asymptotes of \( f(x) = \frac{3x + 3}{x + 2} \). Then graph.

   Zero: \( -1 \); asymptotes: \( x = -2 \), \( y = 3 \)

Solve each equation.

10. \( 2 + \frac{3}{x - 5} = 10 \)

11. \( \frac{x}{x + 1} + \frac{x}{x + 3} = \frac{5}{x + 1} - 5 \)

12. Both can tile a floor in about 6 h. When both Mike and Bob work together, they can tile a floor in about 2.4 h. About how long would it take Mike to tile a floor by himself? 3 h.

Simplify each expression. Assume that all variables are positive.

13. \( \sqrt[5]{-32x^6} \cdot 2x^2(\sqrt{x}) \)

14. \( 8 - \frac{4}{x} \)

15. \( \frac{27}{2^x} \)

16. Write the expression \( \sqrt{x^2} \) by using a rational exponent, \( x^{n/k} \).

17. Graph the function \( f(x) = \sqrt{x - 2} \) and identify its domain and range.

18. Graph the inequality \( y \leq \sqrt{x} - 2 \).

Solve each equation.

19. \( \sqrt{x + 7} = 5 \)

20. \( \sqrt{2x + 1} = \sqrt{x + 9} \)

21. \( (3x + 1)^3 = -2 - 3 \)

22. The formula \( s = \sqrt{\frac{1}{4}A} \) can be used to approximate the side length \( s \) of a regular octagon with area \( A \). A stop sign is shaped like a regular octagon with a side length of 12.4 in. To the nearest square inch, what is the area of the stop sign? 742 in²

23. Solve the inequality \( 2x + 1 > 3 \), \( x > 4 \).
1. A mail employee is dressing a mannequin. There are 6 pairs of shoes, 4 types of jeans, and 8 shirts. Using 1 of each, how many ways can the mannequin be dressed? 192
2. How many ways can you award first, second, and third place to 8 contestants? 336
3. How many ways can a group of 3 students be chosen from a class of 30? 4060
4. Four cards are randomly selected from a standard deck of 52 playing cards. What is the probability that the cards are all jacks, all queens, or all kings? 3/10
5. The table shows the results of tossing 2 coins. Find the experimental probability of tossing 2 tails.

<table>
<thead>
<tr>
<th>HH</th>
<th>HT</th>
<th>TH</th>
<th>TT</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>6</td>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>

Each letter of the alphabet is written on a card. The cards are placed into a bag. Determine whether the events are independent or dependent, and find the indicated probability.

6. The letter $D$ is drawn, replaced in the bag, and then the letter $J$ is drawn. Independent; $\frac{1}{266}$
7. Three streets are drawn without replacement. Dependent; $\frac{1}{256}$

A card is drawn from a bag containing the 9 cards shown. Find each probability.
8. Selecting a C or an even number. $\frac{3}{4}$
9. Selecting an odd number or a multiple of 3. $\frac{4}{9}$
10. The probability distribution for the number of absent students on any given day for a certain class is given. Find the expected number of absent students. $1.3$

A school is voting on a new mascot. While voting, the student body also tracked whether each person played on one of the school's sports teams. The data are presented in the table.

<table>
<thead>
<tr>
<th>Mascot</th>
<th>Plays a sport</th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aardvark</td>
<td>5</td>
<td>75</td>
<td></td>
</tr>
<tr>
<td>Fruit bat</td>
<td>15</td>
<td>35</td>
<td></td>
</tr>
<tr>
<td>Plankton</td>
<td>11</td>
<td>123</td>
<td></td>
</tr>
</tbody>
</table>

11. Make a table of the joint relative frequencies and marginal relative frequencies.
12. Given that a student voted for fruit bat, what is the probability they play a sport? $0.385$
13. Given that a student plays a sport, what is the probability they voted for fruit bat? $0.368$
14. Given that a student plays a sport, what is the probability they did not vote for fruit bat? $0.632$
The number of natural satellites of major objects in the solar system (as of 2005) is given.

<table>
<thead>
<tr>
<th>Planet</th>
<th>Number of Satellites</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>0</td>
</tr>
<tr>
<td>Venus</td>
<td>0</td>
</tr>
<tr>
<td>Earth</td>
<td>1</td>
</tr>
<tr>
<td>Mars</td>
<td>2</td>
</tr>
<tr>
<td>Jupiter</td>
<td>63</td>
</tr>
<tr>
<td>Saturn</td>
<td>33</td>
</tr>
<tr>
<td>Uranus</td>
<td>22</td>
</tr>
<tr>
<td>Neptune</td>
<td>13</td>
</tr>
<tr>
<td>Pluto</td>
<td>1</td>
</tr>
</tbody>
</table>

Source: NASA Planetary Census System, 2005

1. Make a box-and-whisker plot of the data. Find the interquartile range.
2. Is 63 an outlier? Explain.
3. Identify the outlier in the following data set: 93, 107, 110, 105, 98, 95, 12, 111, 128, 99, 114, and 90. Describe how the outlier affects the mean and the standard deviation.
4. Use the binomial theorem to expand \((x + y)^3\). \(81x^3 + 108xy^2 + 54y^3 + 12xy^2 + y^3\)

The probability of winning a carnival game is 15%. Elaine plays 10 times.
5. Find the probability that Elaine will win 2 times. \(= 0.23\)
6. Find the probability that Elaine will win at least 2 times. \(= 0.46\)

7. In a survey of 30 employees at a chain of bookstores, 5 employees said that they had read fewer than 10 books in the last year. The chain has 372 employees. Predict the number of employees at the chain of bookstores who had read fewer than 10 books in the last year. \(62\)

Explain whether the research topic is best addressed through an experiment or an observational study.
8. Does painting without a painter's mask put people at greater risk of respiratory problems? Since it would be unethical to risk the respiratory systems of participants in an experiment, an observational study would be the best way to study this topic.
9. Does using an exercise machine sold on television improve muscle growth? Since the machine is unlikely to cause any damage, an experiment is acceptable.
10. An investment consultation firm claims that it will increase the value of its clients’ investments by 17%. In a random sample of 50 clients of the firm, the average return on investments with minimal risk was 16.1% of the original investment, with a standard deviation of 3%. What is the z-score rounded to the nearest hundredth, and is there enough evidence to reject the firm’s claim? \(-2.12\); yes, the claim can be rejected because the z-score is less than \(-1.96\)
11. The owner of a plumbing company wants to know if his clients are satisfied with the company’s service. He decides to ask 20 clients, randomly chosen from a list of his 43 clients, for their opinions of the company’s service. What kind of sample is this? simple random
12. If \(x\) is a normally distributed random variable with mean \(\mu\) and standard deviation \(\sigma\), then what is the probability that \(x < \mu + \sigma\) \(0.6826\)

Dollie writes a computer subroutine to randomly output a number from an input list. She uses the list 1, 1, -1, 4, -2, -6, 13, -4, 5. What is the expected value of the subroutine’s output? \(0.6785\)
CHAPTER 9

CHAPTER TEST

Find the first 5 terms of each sequence.
1. \( a_n = n^2 - 4 = -3, 0, 5, 12, 21 \)
2. \( a_1 = 48 \) and \( a_n = \frac{1}{2}a_{n-1} - 8 \)
   \( 48, 16, 0, -8, -12 \)

Write a possible explicit rule for the \( nth \) term of each sequence.
3. \( -4, -2, 0, 2, 4, \ldots \quad a_n = 2n - 6 \)
4. \( 54, 18, 6, 2, \frac{2}{3}, \ldots \quad a_n = 54 \left( \frac{1}{3} \right)^{n-1} \)

Expand each series and evaluate.
5. \( \sum_{k=1}^{4} 5k^2 + 40 + 135 + 320 = 500 \)
6. \( \sum_{k=1}^{4} \frac{(-1)^{k+1}}{k} = 1 - 2 + 3 - 4 \)

Find the 9th term of each arithmetic sequence.
7. \( -19, -13, -7, -1, \ldots -29 \)
8. \( a_2 = 11.6 \) and \( a_5 = 5 - 3.8 \)
9. Find 2 missing terms in the arithmetic sequence \( 125, 130, \ldots, 65. \quad 105, 85 \)

Find the indicated sum for each arithmetic series.
10. \( S_n \) for \( 4 + 7 + 10 + 13 + \ldots + 650 \)
11. \( \sum_{k=1}^{12} (9k + 8) = 600 \)

12. The front row of a theater has 16 seats and each subsequent row has 2 more seats than the row that precedes it. How many seats are in the 12th row? How many seats in total are in the first 12 rows? \( 38 \) seats; \( 324 \) seats

Find the 10th term of each geometric sequence.
13. \( \frac{3}{256}, \frac{3}{16}, \frac{3}{4}, \ldots \quad 3072 \)
14. \( a_2 = 2 \) and \( a_5 = 8 \)
15. Find the geometric mean of \( 4 \) and \( 23. \quad 10 \)

Find the indicated sum for each geometric series.
16. \( S_n \) for \( 2 + 1 + \frac{1}{2} + \frac{1}{4} + \ldots + \frac{63}{16} \)
17. \( \sum_{k=1}^{5} 250 \left( -\frac{1}{5} \right)^{k-1} = 208.32 \)

18. You invest \( $1000 \) each year in an account that pays \( 5\% \) annual interest. How much is the first \( $1000 \) you invested worth after 10 full years of interest payments? How much is the total of all of your accounts worth after 10 full years? \( $1628.89; $13,206.79 \)

Find the sum of each infinite geometric series, if it exists.
19. \( 200 - 100 + 50 - 25 + \ldots \quad \frac{400}{3} \) or \( 133.\overline{3} \)
20. \( \sum_{k=1}^{\infty} \frac{7}{8} \)

20. Use mathematical induction to prove \( \frac{1}{2} + \frac{3}{2} + \frac{5}{2} + \ldots + \frac{2n-1}{2} = \frac{n^2}{2} \).
21. Step 1
22. Step 2
23. Step 3

Answers
21. \( \frac{2(1)-1}{2} = \frac{2}{2} = \frac{1}{2} \)
22. \( \frac{1}{2} + \ldots + \frac{2k-1}{2} = \frac{k^2}{2} \)
23. \( \frac{1}{2} + \ldots + \frac{2k-1}{2} + \frac{2(k+1)-1}{2} \)

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Find the values of the six trigonometric functions for $\theta$.

1. \[ \sin \theta = \frac{3}{5}, \quad \cos \theta = \frac{4}{5}, \quad \tan \theta = -\frac{3}{4} \]
   \[ \csc \theta = \frac{5}{3}, \quad \sec \theta = -\frac{5}{4}, \quad \cot \theta = -\frac{4}{3} \]

2. \[ \sin \theta = \frac{5}{3}, \quad \cos \theta = -\frac{2}{3}, \quad \tan \theta = -\frac{5}{2} \]
   \[ \csc \theta = \frac{3}{5}, \quad \sec \theta = -\frac{3}{2}, \quad \cot \theta = -\frac{2}{5} \]

3. Katrina is flying a kite on 150 ft of string. The string makes an angle of $62^\circ$ with the horizontal. If Katrina holds the end of the string 5 ft above the ground, how high is the kite? Round to the nearest foot. 137 ft

Draw an angle with the given measure in standard position.

4. $100^\circ$  
5. $-210^\circ$

$P$ is a point on the terminal side of $\theta$ in standard position. Find the exact value of the six trigonometric functions of $\theta$.

6. $P(-32, 24)$  
7. $P(-3, -7)$

Convert each measure from degrees to radians or from radians to degrees.

8. $310^\circ \rightarrow \frac{31\pi}{18}$ radians  
9. $-36^\circ \rightarrow -\frac{\pi}{5}$ radians  
10. $\frac{2\pi}{9} \rightarrow 40^\circ$  
11. $\frac{5\pi}{6} \rightarrow -150^\circ$

Use the unit circle to find the exact value of each trigonometric function.

12. $\cos 210^\circ = -\frac{\sqrt{3}}{2}$  
13. $\tan \frac{11\pi}{6} = \frac{\sqrt{3}}{3}$

Evaluate each inverse trigonometric function. Give your answer in both radians and degrees.

14. $\cos^{-1} \left( \frac{\sqrt{3}}{2} \right) \rightarrow \frac{\pi}{4}$ radians; $45^\circ$  
15. $\sin^{-1} \left( -\frac{\sqrt{3}}{2} \right) \rightarrow -\frac{\pi}{3}$ radians; $-60^\circ$

Evaluate each inverse trigonometric function. Give your answer in both radians and degrees.

14. $\cos^{-1} \left( \frac{\sqrt{3}}{2} \right) \rightarrow 45^\circ$  
15. $\sin^{-1} \left( -\frac{\sqrt{3}}{2} \right) \rightarrow -60^\circ$

16. A lifeguards cage is 62 mi south and 14 mi east of the entrance of a national park. To the nearest degree, in what direction should a group at the entrance head in order to reach the cage? $77^\circ$ south of east

17. Find the area of $\triangle DEF$. Round to the nearest tenth. $425.8$ sq cm

18. Use the given measurements to solve $\triangle DEF$. Round to the nearest tenth. $m_\angle D = 29.2^\circ$, $m_\angle F = 71.8^\circ$, $e = 44.8$ cm

19. An artist is designing a wallpaper pattern based on a triangle. Determine the number of different triangles she can form using the measurements $a = 26$, $b = 13$, and $m_\angle A = 102^\circ$. Then solve the triangle. Round to the nearest tenth. 1 triangle; $m_\angle B = 27.9^\circ$, $m_\angle C = 51.0^\circ$, $c = 22.2$ cm

20. Solve $\triangle ABC$. Round to the nearest tenth. $m_\angle A = 100^\circ$, $b = 12.0$, $m = 10.1$

21. A lawn next to an office building is shaped like a triangle with sides measuring 16 ft, 24 ft, and 30 ft. What is the area of the lawn to the nearest square foot? $191$ ft

Answers

4. \[ \sin \theta = \frac{3}{5}, \quad \cos \theta = \frac{4}{5}, \quad \tan \theta = -\frac{3}{4} \]
   \[ \csc \theta = \frac{5}{3}, \quad \sec \theta = -\frac{5}{4}, \quad \cot \theta = -\frac{4}{3} \]

6. $\sin \theta = \frac{3}{5}, \quad \cos \theta = -\frac{4}{5}, \quad \tan \theta = -\frac{3}{4}$  
   $\csc \theta = \frac{5}{3}, \quad \sec \theta = -\frac{5}{4}, \quad \cot \theta = -\frac{4}{3}$

7. $\sin \theta = \frac{7\sqrt{3}}{58}$, $\cos \theta = -\frac{3\sqrt{3}}{58}$  
   $\tan \theta = \frac{7}{3}$  
   $\csc \theta = -\frac{58}{7\sqrt{3}}$, $\sec \theta = -\frac{58}{3}$  
   $\cot \theta = \frac{3}{7}$

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