Ellipses

Lesson 10-3
**Ellipses**

**Ellipse:** the set of all points in a plane, the sum of whose distances from two fixed points, called foci, is constant.

Creating your own ellipse is easy if you can affix a loose string at both ends and use a pencil.
The center has coordinates \((h, k)\)
- 2 axes of symmetry ... longest is major axis, shortest is minor axis.
- Ellipse has 4 vertices – the endpoints of each of the axes.
- Distance from center along semi-major axis to vertex is “a.”
- Distance from center along semi-minor axis is “b.”
- Distance from center to either foci is “c.”
- For all ellipses, \(a^2 - b^2 = c^2\)
- The measure of distortion from pure circularity is called eccentricity \((e)\) and the equation \(e = c/a\) (as \(e\) approaches 0, more circular, as \(e\) approaches 1, greater distortion)
The Standard Form of the equation of an ellipse.

\[
\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1
\]

This ellipse is horizontally oriented ... “a” is the largest segment and it is the denominator of the “x” term.

\[
\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1
\]

This ellipse is vertically oriented ... “a” is the largest segment and it is the denominator of the “y” term.
For the ellipse \( \frac{(x - 3)^2}{25} + \frac{(y + 2)^2}{16} = 1 \), find the center, vertices, foci, and eccentricity.

Orientation: horizontal. Largest denominator under the “x” term.

Center: (3, -2) pull directly from the equation.

Vertices: to find these we need the distances labeled “a” and “b.” These are the square roots of the denominators.

\[ a^2 = 25, \text{ so therefore } a = 5 \]
\[ b^2 = 16, \text{ so therefore } b = 4 \]

Vertices are: (-2, -2) (8, -2) (3, 2) (3, -6)

Foci: these are a distance “c” from the center along the major axis.

Remember: \( a^2 - b^2 = c^2 \) ... By substitution we have

\[ 25 - 16 = c^2 \]
\[ 9 = c^2 \]
\[ 3 = c \]

Foci are located 3 units to either side of the center: (0, -2) & (6, -2)
Find the orientation, center, foci, vertices, eccentricity of each ellipse.

\[
\frac{(y - 2)^2}{169} + \frac{(x - 5)^2}{144} = 1
\]

**Orientation:** vertical

**Center:** (5, 2)

\(a = 13\) and \(b = 12\)

**Vertices:** (5, 15) (5, -11) (17, 2) (-7, 2)

\(c = 5\)

**Foci:** (5, 7) and (5, -3)

\(e = c/a = 5/13\)
**The Summary:**

<table>
<thead>
<tr>
<th>Standard Form of the Equation of an Ellipse</th>
<th>Orientation</th>
<th>Description</th>
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</table>
| \( \frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \), where \( c^2 = a^2 - b^2 \) | ![Orientation Diagram 1](image1.png) | Center: \((h, k)\)  
Foci: \((h \pm c, k)\)  
Major axis: \(y = k\)  
Major axis vertices: \((h \pm a, k)\)  
Minor axis: \(x = h\)  
Minor axis vertices: \((h, k \pm b)\) |
| \( \frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1 \), where \( c^2 = a^2 - b^2 \) | ![Orientation Diagram 2](image2.png) | Center: \((h, k)\)  
Foci: \((h, k \pm c)\)  
Major axis: \(x = h\)  
Major axis vertices: \((h, k \pm a)\)  
Minor axis: \(y = k\)  
Minor axis vertices: \((h \pm b, k)\) |