Chapter 10

Sequences and Series
Chapter 10 Agenda:
Arithmetic Sequences
Arithmetic Series
Geometric Sequences
Geometric Series
Infinite Geometric Series
Recursive Formulas
Fibonacci and Other Special Sequences
Arithmetic Sequences
**Arithmetic Sequence** - a list of numbers that have a common difference between terms

Find the next three terms:

1, 4, 7, 10, . . .

5, 11, 17, 23, . . .

9, 5, 1, -3, . . .

$\frac{1}{2}, 3, \frac{11}{2}, 8, . . .$

To find the common difference "d", subtract the first term from the second term. The difference between successive terms should be the same for the entire sequence.
Arithmetic Sequences

Find d:

1, 4, 7, 10, . . .

5, 11, 17, 23, . . .

9, 5, 1, -3, . . .

$\frac{1}{2}, 3, \frac{11}{2}, 8, . . .
1 Find the next term in the arithmetic sequence:
3, 9, 15, 21, . . .
2. Find the next term in the arithmetic sequence:
   -8, -4, 0, 4, . . .
3 Find the next term in the arithmetic sequence:
2.3, 4.5, 6.7, 8.9, ...
4 Find the value of d in the arithmetic sequence:
10, -2, -14, -26, . . .
5 Find the value of $d$ in the arithmetic sequence:
-8, 3, 14, 25, . . .
Arithmetic Sequences

As we study sequences we need a way of naming the terms:

- $a_1$ represents the first term,
- $a_2$ represents the second term,
- $a_3$ represents the third term,
- and so on in this manner.

If we were talking about the 8th term we would use $a_8$.

When we want to represent any random term, call it the $n$th term and use $a_n$. 
**Arithmetic Sequence**

Write the next three terms of the arithmetic sequence that is described.

\[ a_1 = 4; d = 6 \]

\[ a_1 = 3; d = -3 \]

\[ a_1 = 0.5; d = 2.3 \]

\[ a_2 \quad a_3 \quad a_4 \]

\[ a_1 = 7; d = 5 \]
6 Which sequence matches the description?

\[ a_1 = 2; d = 4 \]

A  4, 6, 8, 10
B  2, 6, 10, 14
C  2, 8, 32, 128
D  4, 8, 16, 32
Which sequence matches the description? 

\[ a_1 = -3; d = -4 \]

A. -3, -7, -10, -14
B. -4, -7, -11, -13
C. -3, -7, -11, -15
D. -3, 1, 5, 9
8 Which sequence matches the description? 

\[ a_3 = 7; d = 3 \]

A 7, 10, 13, 16
B 4, 7, 10, 13
C 1, 4, 7, 10
D 3, 5, 7, 9
Arithmetic Sequences

To find a specific term, say the $5^{th}$ or $a_5$, you could write out all of the terms fairly quickly.

But what about the $100^{th}$ term (or $a_{100}$)?

We need to find a formula to arrive at the answer quickly without writing out the whole list.
Arithmetic Sequence

Consider: 3, 9, 15, 21, 27, 33, 39, . . .

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a1</td>
<td>3</td>
</tr>
<tr>
<td>a2</td>
<td>9 = 3+6</td>
</tr>
<tr>
<td>a3</td>
<td>15 = 3+12 = 3+2(6)</td>
</tr>
<tr>
<td>a4</td>
<td>21 = 3+18 = 3+3(6)</td>
</tr>
<tr>
<td>a5</td>
<td>27 = 3+24 = 3+4(6)</td>
</tr>
<tr>
<td>a6</td>
<td>33 = 3+30 = 3+5(6)</td>
</tr>
<tr>
<td>a7</td>
<td>39 = 3+36 = 3+6(6)</td>
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</tbody>
</table>

Do you see a pattern that relates the term number to its value?

\[ a_n = a_1 + (n-1)d \]
**Arithmetic Sequences**

Example Find the 21\textsuperscript{st} term of the arithmetic sequence with \(a_1 = 4\) and \(d = 3\).

Example Find the 12\textsuperscript{th} term of the arithmetic sequence with \(a_1 = 6\) and \(d = -5\).
**Arithmetic Sequences**

**Example** Find the 15th term of the arithmetic sequence with \(a_{15} = 30\) and \(d = 7\).

**Example** Find the 17th term of the arithmetic sequence with \(a_{17} = 4\) and \(d = -2\).
Arithmetic Sequences

Example Find "d" of the arithmetic sequence with \( a_{15} = 42 \) and \( a_1 = 3 \).

Example Find the term number \( n \) of the arithmetic sequence with \( a_1 = 6 \), \( a_1 = -34 \) and \( d = 4 \).
9. Find $a_{11}$ when $a_1 = 13$ and $d = 6$. 
10 Find $a_{17}$ when $a_1 = 12$ and $d = -0.5$
Find $a_{17}$ for the sequence 2, 4.5, 7, 9.5, ...
Find the common difference \( d \) when \( a_1 = 12 \) and \( a_{14} = 6 \).
13. Find $n$ such $a_1 = 12$, $a_n = -20$, and $d = -2$. 
Arithmetic Sequences

Find the missing terms

2, ____, ____, 23

4, ____, ____ , -14

7, ____, ____ , ____ , 39

-9, ____ , ____ , ____ , ____ , -34
14 Find the missing term: 4, ___ , -16

A  -20
B  -10
C  -6
D  2
15 Find the missing terms: -10, ____ , ____ , 8

A  -6, -2
B  -6, 2
C  -5, 1
D  -4, 2
16  Find the missing terms:  12, ___, ___, 75

A  27, 51
B  33, 54
C  37, 51
D  34, 58
17 Find \( d \) for the arithmetic sequence:
5, \_
\,
\_
\,
\_
\,
\_
\,
22