A polygon is a plane figure that is formed by three or more segments called sides. The endpoint of each side is a vertex.

A segment that joins two nonconsecutive vertices of a polygon is called a diagonal.

Polygons are classified by the number of sides they have. A triangle has three sides. A quadrilateral has four sides. A pentagon has five sides. A hexagon has six sides. A heptagon has seven sides. An octagon has eight sides.

Theorem 6.1 Quadrilateral Interior Angles Theorem
The sum of the measures of the interior angles of a quadrilateral is 360°.

Tell whether the figure is a polygon. Explain your reasoning.

a. No, the figure is not a polygon because each side intersects two other sides at one vertex, and no other sides at the other vertex.

b. No, the figure is not a polygon because it has a side that is not a segment.

c. Yes, the figure is a polygon formed by five straight sides.

Exercises for Example 1
Tell whether the figure is a polygon. Explain your reasoning.

1. 
2. 
3. 
4. 
5.
**Example 2**

Classify Polygons

Decide whether the figure is a polygon. If so, tell what type. If not, explain why.

a. The figure is a polygon with seven sides, so it is a heptagon.

b. The figure is not a polygon because two of the sides intersect only one other side.

c. The figure is a polygon with six sides, so it is a hexagon.

**Exercises for Example 2**

Decide whether the figure is a polygon. If so, tell what type. If not, explain why.

6. 

7. 

8. 

**Example 3**

Use the Quadrilateral Interior Angles Theorem

Find the value of $x$.

**Solution**

Use the fact that the sum of the measures of the interior angles of a quadrilateral is $360^\circ$.

\[
\begin{align*}
50^\circ + 118^\circ + 84^\circ + 2x^\circ &= 360^\circ \\
2x &= 108 \\
x &= 54
\end{align*}
\]

**Exercises for Example 3**

Find the value of $x$.

9. 

10. 

11. 

---

*Geometry*

Chapter 6 Resource Book

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**Goal:** Use properties of parallelograms.

**Vocabulary**
A parallelogram is a quadrilateral with both pairs of opposite sides parallel.

**Theorem 6.2**
If a quadrilateral is a parallelogram, then its opposite sides are congruent.

**Theorem 6.3**
If a quadrilateral is a parallelogram, then its opposite angles are congruent.

**Theorem 6.4**
If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

**Theorem 6.5**
If a quadrilateral is a parallelogram, then its diagonals bisect each other.

---

**Example 1: Find Side Lengths of Parallelograms**

$ABCD$ is a parallelogram.

Find the values of $x$ and $y$.

**Solution**

$AB = CD$  
$3x = 15$  
$x = 5$

$BC = AD$  
$y - 5 = 39$  
$y = 44$

---

**Exercises for Example 1**

Find the values of $x$ and $y$ in the parallelogram.

1. 2. 3.
Write the statement of each theorem in symbols for \( \square PQRS \), where \( m\angle SPQ = m\angle QRS = x^\circ \) and \( m\angle RSP = m\angle PQR = y^\circ \).

1. If a quadrilateral is a parallelogram, then its opposite sides are congruent.
2. If a quadrilateral is a parallelogram, then its opposite angles are congruent.
3. If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.
4. If a quadrilateral is a parallelogram, then its diagonals bisect each other.

Find the lengths or angle measures.

5. Find \( DA \) and \( DC \).
6. Find \( GE \) and \( DF \).
7. Find \( m\angle P \), \( m\angle Q \), and \( m\angle R \).
8. Find \( GE \) and \( GF \).
9. Find \( DA \) and \( DC \).
10. Find \( m\angle Q \), \( m\angle R \), and \( m\angle S \).

Find the values of \( x \) and \( y \) in the parallelogram.

11. \[ \begin{align*}
\text{\( y^\circ \)} & \quad \text{\( x^\circ \)} & \quad \text{131}^\circ \\
\end{align*} \]
12. \[ \begin{align*}
5y - 3 & \quad 2x + 1 \\
5 & \quad 12 \\
\end{align*} \]
13. \[ \begin{align*}
3x - 1 & \quad 4y + 4 \\
16 & \quad 14 \\
\end{align*} \]

The chevron symbol shown at the right is used to direct traffic flow.

14. For \( \square JKPM \), name two pairs of congruent sides.
15. For \( \square MPQN \), name two pairs of congruent angles.
16. For \( \square JKPM \), name two angles that are supplementary to \( \angle K \).
Show that a quadrilateral is a parallelogram.

**Vocabulary**

Theorem 6.6
If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Theorem 6.7
If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Theorem 6.8
If an angle of a quadrilateral is supplementary to both of its consecutive angles, then the quadrilateral is a parallelogram.

Theorem 6.9
If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

**Example 1**

**Use Opposite Sides**

Tell whether the quadrilateral is a parallelogram. Explain your reasoning.

a.  

b.  

**Solution**

a. The quadrilateral is a parallelogram because both pairs of opposite sides are congruent.

b. The quadrilateral is not a parallelogram. Both pairs of opposite sides are not congruent.

**Exercises for Example 1**

Tell whether the quadrilateral is a parallelogram. Explain your reasoning.

1.  

2.  

3.  

---

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**Example 2**

**Use Opposite Angles**

Tell whether the quadrilateral is a parallelogram. Explain your reasoning.

a. The quadrilateral is not a parallelogram. Both pairs of opposite angles are not congruent.

b. The quadrilateral is a parallelogram because both pairs of opposite angles are congruent.

**Exercises for Example 2**

Tell whether the quadrilateral is a parallelogram. Explain your reasoning.

4. 

5. 

6. 

**Example 3**

**Use Diagonals and Consecutive Angles**

Tell whether the quadrilateral is a parallelogram. Explain your reasoning.

a. The diagonals of $ABCD$ bisect each other. So, by Theorem 6.9, $ABCD$ is a parallelogram.

b. $\angle H$ is supplementary to $\angle E$ and $\angle G$ ($87^\circ + 93^\circ = 180^\circ$). So, by Theorem 6.8, $EFGH$ is a parallelogram.

**Exercises for Example 3**

Tell whether the quadrilateral is a parallelogram. Explain your reasoning.

7. 

8. 

9. 

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A rhombus is a parallelogram with four congruent sides.

A rectangle is a parallelogram with four right angles.

A square is a parallelogram with four congruent sides and four right angles.

Rhombus Corollary
If a quadrilateral has four congruent sides, then it is a rhombus.

Rectangle Corollary
If a quadrilateral has four right angles, then it is a rectangle.

Square Corollary
If a quadrilateral has four congruent sides and four right angles, then it is a square.

Theorem 6.10
The diagonals of a rhombus are perpendicular.

Theorem 6.11
The diagonals of a rectangle are congruent.

EXAMPLE 1
Use Properties of Special Parallelograms

In the diagram, $ABCD$ is a square.

Find the values of $x$ and $y$.

**Solution**

By definition, a square has four right angles.

$m \angle A = 90^\circ$  

$10x^\circ = 90^\circ$  

$x = 9$  

By definition, a square has four congruent sides. So, $AB = BC$.

$5 = y - 3$  

$8 = y$  

Add 3 to each side.

Exercises for Example 1

Find the values of the variables.

1. rectangle $ABCD$  

2. rhombus $EFGH$  

3. square $JKLM$
**EXAMPLE 2**

**Use Diagonals of a Rhombus**

**JKLM** is a rhombus.
Find the value of $x$.

**SOLUTION**

By Theorem 6.10, the diagonals of a rhombus are perpendicular. Therefore, $\angle KNL$ is a right angle, so $\triangle KNL$ is a right triangle.

By the Corollary to the Triangle Sum Theorem, the acute angles of a right triangle are complementary.

\[
m\angle NKL + m\angle KLN = 90^\circ
\]

Substitute $55^\circ$ for $m\angle NKL$ and $7x^\circ$ for $m\angle KLN$.

\[
55^\circ + 7x^\circ = 90^\circ
\]

Subtract 55 from each side.

\[
7x = 35
\]

Divide each side by 7.

\[
x = 5
\]

**Exercises for Example 2**

Find the value of $x$.

4. rhombus $ABCD$

5. rhombus $EFGH$

**EXAMPLE 3**

**Use Diagonals of a Rectangle**

$ABCD$ is a rectangle. $AC = 16$
$BD = 5x + 1$. Find the value of $x$.

**SOLUTION**

By Theorem 6.11, the diagonals of a rectangle are congruent. Therefore, $AC = BD$.

\[
16 = 5x + 1
\]

Subtract 1 from each side.

\[
15 = 5x
\]

Divide each side by 5.

\[
x = 3
\]

**Exercises for Example 3**

Find the value of $x$.

6. rectangle $EFGH$, $EG = 48$, $HF = 6x$

7. rectangle $WXYZ$, $XZ = 37$, $WY = 5x + 2$
Use properties of trapezoids.

**Vocabulary**

A trapezoid is a quadrilateral with exactly one pair of parallel sides. The parallel sides are called the bases. The nonparallel sides are called the legs.

A trapezoid has two pairs of base angles.

If the legs of a trapezoid are congruent, then the trapezoid is an isosceles trapezoid.

The midsegment of a trapezoid is the segment that connects the midpoints of its legs.

**Theorem 6.12**

If a trapezoid is isosceles, then each pair of base angles is congruent.

**Theorem 6.13**

If a trapezoid has a pair of congruent base angles, then it is isosceles.

**Example 1**

Find Angle Measures of Trapezoids

Find the missing angle measures.

**Solution**

By definition, a trapezoid has exactly one pair of parallel sides. In trapezoid $ABCD$, $AB \parallel CD$. Because $\angle A$ and $\angle D$ are same-side interior angles formed by parallel lines, they are supplementary. So, $m\angle A = 180^\circ - m\angle D = 180^\circ - 82^\circ = 98^\circ$.

Because $\angle B$ and $\angle C$ are same-side interior angles formed by parallel lines, they are supplementary. So, $m\angle B = 180^\circ - m\angle C = 180^\circ - 75^\circ = 105^\circ$.

**Exercises for Example 1**

$EFGH$ is a trapezoid. Find the missing angle measures.

1. $\angle G = 45^\circ$
2. $\angle F = 122^\circ$
3. $\angle H = 72^\circ$
Using Theorem 6.12

ABCD is an isosceles trapezoid. Find the values of x and y.

**SOLUTION**

By Theorem 6.12, each pair of base angles in an isosceles trapezoid is congruent. In trapezoid ABCD, \( \angle A \) and \( \angle D \) are a pair of base angles, and \( \angle B \) and \( \angle C \) are a pair of base angles.

\[
m\angle A = m\angle D \\
132^\circ = 12x^\circ \\
11 = x
\]

Divide each side by 12.

\[
m\angle B = m\angle C \\
(5y - 2)^\circ = 48^\circ \\
5y = 50 \\
y = 10
\]

Add 2 to each side.

Divide each side by 5.

**Exercises for Example 2**

Find the values of the variables.

4. isosceles trapezoid EFGH

5. isosceles trapezoid JKL

---

**Example 3**

Midsegment of a Trapezoid

Find the length of the midsegment \( AB \) of trapezoid JKL.

**SOLUTION**

Use the formula for the midsegment of a trapezoid.

\[
AB = \frac{1}{2}(JK + LM)
\]

Formula for midsegment of a trapezoid

\[
AB = \frac{1}{2}(17 + 13)
\]

Substitute 17 for \( JK \) and 13 for \( LM \).

\[
AB = 15
\]

Simplify.

**Exercises for Example 3**

Find the length of the midsegment \( AB \) of the trapezoid.
Identify special quadrilaterals based on limited information.

**EXAMPLE 1**

**Use Properties of Quadrilaterals**

Determine whether the quadrilateral is a trapezoid, isosceles trapezoid, parallelogram, rectangle, rhombus, or square.

**a.** The diagram shows that \( \angle A \) is supplementary to \( \angle B \) and to \( \angle D \). Since one angle of the quadrilateral is supplementary to both of its consecutive angles, you know that \( ABCD \) is a parallelogram by Theorem 6.8.

**b.** The diagram shows that all four sides of quadrilateral \( EFGH \) have length 8. Since all four sides are congruent, \( EFGH \) is a rhombus.

**Exercises for Example 1**

Determine whether the quadrilateral is a trapezoid, isosceles trapezoid, parallelogram, rectangle, rhombus, or square.

1. 

2. 

3. 

4. 

5. 

6. 
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For use with pages 337–341

**EXAMPLE 2**

Identify a Quadrilateral

Are you given enough information to conclude that the figure is the given type of special quadrilateral? Explain your reasoning.

a. A rectangle?
   
   ![Rectangle Diagram]
   
   **SOLUTION**
   
   a. The diagram shows that both pairs of opposite sides are congruent. Therefore, you know that \(ABCD\) is a parallelogram. For \(ABCD\) to be a rectangle, all four angles must be right angles. The diagram does not give any information about the angle measures, so you cannot conclude that \(ABCD\) is a rectangle.

b. A square?
   
   ![Square Diagram]
   
   **SOLUTION**
   
   b. The diagram shows that all four sides are congruent. Therefore, you know that \(EFGH\) is a rhombus. For \(EFGH\) to be a square, all four sides must be congruent and all four angles must be right angles. By the Quadrilateral Interior Angles Theorem, you know that the sum of the measures of the four angles must equal 360°. From the diagram, you know that all four angles have the same measure.

   \[
   m\angle E + m\angle F + m\angle G + m\angle H = 360° \\
   x° + x° + x° + x° = 360° \\
   4x = 360
   \]

   **Simplify.**

   \[
   x = 90
   \]

   Divide each side by 4.

   Because all four angles are right angles and all four sides are congruent, you know that \(EFGH\) is a square.

c. A parallelogram?
   
   ![Parallelogram Diagram]
   
   **SOLUTION**
   
   c. The diagram shows that one pair of opposite sides is parallel and the one pair of consecutive angles is supplementary (132° + 48° = 180°). There is no information given about the second pair of opposite sides, nor is there any information given about any other pair of consecutive angles. Therefore, you cannot conclude that \(JKLM\) is a parallelogram.

**Exercises for Example 2**

Are you given enough information to conclude that the figure is the given type of special quadrilateral? Explain your reasoning.

7. An isosceles trapezoid?
   
   ![Isosceles Trapezoid Diagram]

8. A rhombus?
   
   ![Rhombus Diagram]

9. A parallelogram?